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WALDMEIER LAW FOR SUNSPOT CYCLES: STATISTICAL SIGNIFICANCE AND IMPLICATIONS FOR DYNAMO

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Abstract. The Waldmeier law for the relation between the rise phase duration and amplitude of solar cycles is an indicator for strength of nonlinear effect in the solar dynamo mechanism. The paper compares the Waldmeier law parameters for sunspot numbers and areas in solar cycles 12–25. Uncertainty in dates of cycles' maxima and minima is fixed by smoothing with the Gaussian filter. The ratio of sunspot number to area is shown to differ significantly between rise and decline phases of solar activity. The high correlation for Waldmeier law in sunspot number is an artifact of the Wolf number definition and does not imply any strong nonlinearity in the dynamo mechanism. The relative low correlation and significance level of about 70% in the Waldmeier law for sunspot areas indicates a weak nonlinearity in the solar dynamo and agrees with evidence from observations of stellar rotation.

Keywords: solar activity, sunspot number, sunspot area, dynamo, nonlinearity

INTRODUCTION

Among multiple regularities found for the solar cycles, the Waldmeier law is probably the most significant for the solar dynamo theory. The law states an inverse statistical relation between the solar cycles' amplitude and rise phase duration from minimum to maximum of a cycle: the higher is amplitude, the shorter is the rise phase [Waldmeier, 1935]. The Waldmeier law for the Wolf number of sunspots has a correlation coefficient of about –0.7 and statistical significance above 98 % [Hathaway, 2015]. Such strong dependence on the cycle's amplitude is indicative of a strong nonlinearity in the solar activity mechanism. There is, however, evidence for the opposite.

Magnetic field generation in solar and stellar dynamo models requires the dynamo-number to exceed a certain threshold value [Charbonneau, 2020; Charbonneau, Sokoloff, 2023]. The field amplification is stabilized by a back reaction of the magnetic field to the field-generating flow, i.e., by nonlinear effects. The relative excess of the dynamo number over its threshold value for the onset of the dynamo – the so-called "supercriticality" – may serve as a measure of nonlinearity. The dynamo number can be understood as an appropriately normalized rate of rotation. Stellar observations [van Saders et al., 2016] show a weakening of magnetic activity with decreasing rotation rate. The rotational spindown proceeds until its rate falls to a certain minimum value dependent on the stellar mass. This minimum rotation rate is naturally interpreted as the critical rate for stellar dynamos [Kitchatinov, Nepomnyashchikh, 2017a; Cameron, Schussler, 2017]. Solar rotation rate exceeds this critical value by ~10 % [Metcalfe et al., 2016]. Supercriticality of the solar dynamo can therefore be estimated at ~10 % as well [Kitchatinov, Nepomnyashchikh, 2017b]. Solar dynamo models reproduce the Waldmeier law with much larger supercriticality only [Karak, Choudhuri, 2011; Pipin, Kosovichev, 2011].

Sunspot area is an important parameter of solar activity physics. The area is a measure of magnetic flux in active regions [Tlatov, Pevtsov, 2014]. Solar dynamo parameters can be estimated from sunspot area data [Dasi-Espuig et al., 2010; Kitchatinov, Olemskoy, 2011; Jiang et al., 2013; Olemskoy et al., 2013; Sreedevi et al., 2024].

The first estimation of the Waldmeier effect from sunspot area data by Dikpati et al. [2008] did not, however, reveal any effect of this type. Later on, Karak and Choudhuri [2011] draw attention to the fact that the result obtained by Dikpati et al. can be related to uncertainty in dates of solar maxima and minima for the area data smoothed with Gaussian filter of one year half-width and, if definition of the rise phase duration is changed to resolve the uncertainty, the Waldmeier law is reestablished though at a lower significance level (~67 %) compared to sunspot number data. It will be shown in this paper that the uncertainty in dates can also be resolved by increasing half-width of the filter. In this way, the Waldmeier effect is evidenced by the standard definition of the growth phase duration in accord with the results received by Karak and Choudhuri [2011] and Osipova and Nagovitsyn [2017]. Our primary aim is, however, to find out the reason for the difference in Waldmeier law parameters between sunspot numbers and areas. The difference is probably caused by a nonlinear relation between sunspot area (magnetic flux) and sunspot number [Carrasco et al., 2016; Nagovitsyn, Osipova, 2021], which is rooted in the very definition of the Wolf number. In this case, the relative low correlation coefficient and statistical significance in the Waldmeier law for sunspot areas do not any longer imply a strong nonlinearity for the solar dynamo mechanism.

METHOD

The monthly mean sunspot area data series from May 1874 to December 2025 known as the extended Greenwich Observatory data series (http://solarcyclescience.com/AR_Database/sunspot_area.txt) is used in our study. For sunspot number (version 2.0), we employ a part of the World Data Center for Solar-Terrestrial Physics data series (http://www.wdcb.ru/stp/data/solar.act/sunspot/SILSO/ver2/SN_m/SN_m_tot_V2.0.txt) for the same date range.

Irregular fluctuations in both data series are smoothed out with the Gaussian filter,

$$W(t) = \exp\left(-\frac{t^2}{2a^2}\right) - \left(3 - \frac{t^2}{2a^2}\right)e^{-2}, \quad -2a+1 \le t \le 2a-1, \tag{1}$$

where t is time in months and $2a$ is the filter half-width. The Gaussian filter smoothing differs preferably from the traditional 13-month smoothing according to the absence of smoothing-induced artificial short-period oscillations (see Figure 23 in Hathaway [2015]). Uncertainty in the dates of cycles' minima and maxima disappears for a = 12 months, which is important for the Waldmeier effect analysis. The smoothed sunspot areas and numbers are shown in Figure 1. Only cycle 16 in the smoothed area data has a double peak. The date of maximum is attributed to the higher first peak of this cycle.

Two comments on smoothing with the filter of Eq. (1) need to be explained. A smoothed value for a certain date is a sum of the data values outstanding from the date by several months multiplied by weights of Eq. (1) and divided by the sum of weights over the range of smoothing. In a strict sense, this procedure is applicable only to dates distant by not less than $2a - 1$ months from the edge dates of the data series. Data smoothing near the edge dates is done over those part of filter (1) that covers the dates present in the data series (normalized to the sum of weights over such a 'shortened' range of smoothing). This is why Figure 1 comprises the maximum of cycle 25 in October 2024. Note also that the broader is the range of smoothing, the higher are minima and the lower are maxima in the smoothed series. The cycle amplitudes are, therefore, somewhat smaller compared to the traditional 13-month running mean.

RESULTS AND DISCUSSION

Figure 2 compares the Waldmeier law parameters for sunspot areas and numbers. A considerable scatter of cycle positions can be seen in both plots of the Figure. Irregular variations in cycle amplitudes and durations are caused by either fluctuations in dynamo parameters [Moss et al., 2008; Olemskoy et al., 2013] or observational limitations [Vitinsky et al., 1986].

Correlation between growth time and cycle amplitude is much higher for spot numbers than for their areas. There is also a considerable difference in statistical significance estimated by the false correlation probability,

$$P_{\text{false}} = 1 - A\left(r\sqrt{\frac{n-2}{1-r^2}}\,\middle|\, n-2\right), \tag{2}$$

where $A(t|v)$ is the Student's distribution, r is the correlation coefficient, and n is the number of events (14 cycles in our case).

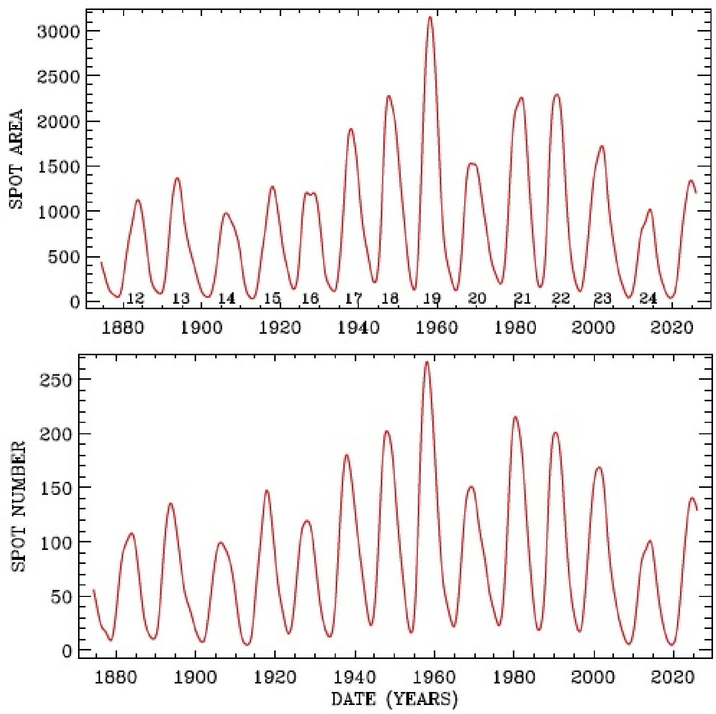

Figure 1. Sunspot area in millionths of the solar hemisphere (top) and sunspot number (bottom) for solar cycles 12−25. The cycles' number is shown in the top panel

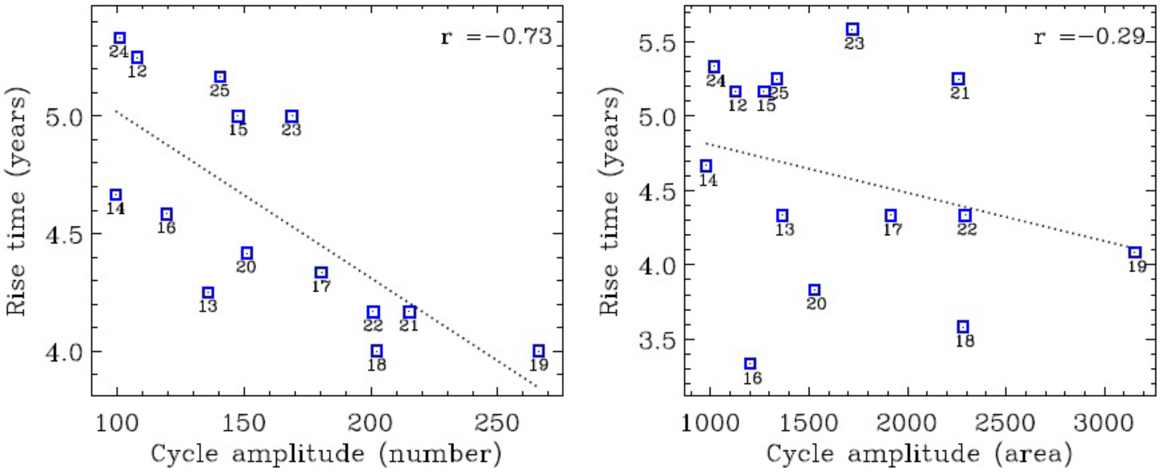

Figure 2. Solar cycle positions on the growth time – amplitude diagram for spot numbers (left panel) and areas (right panel). The linear regression is indicated by dotted lines. Values of the correlation coefficient *r* are given in the plot

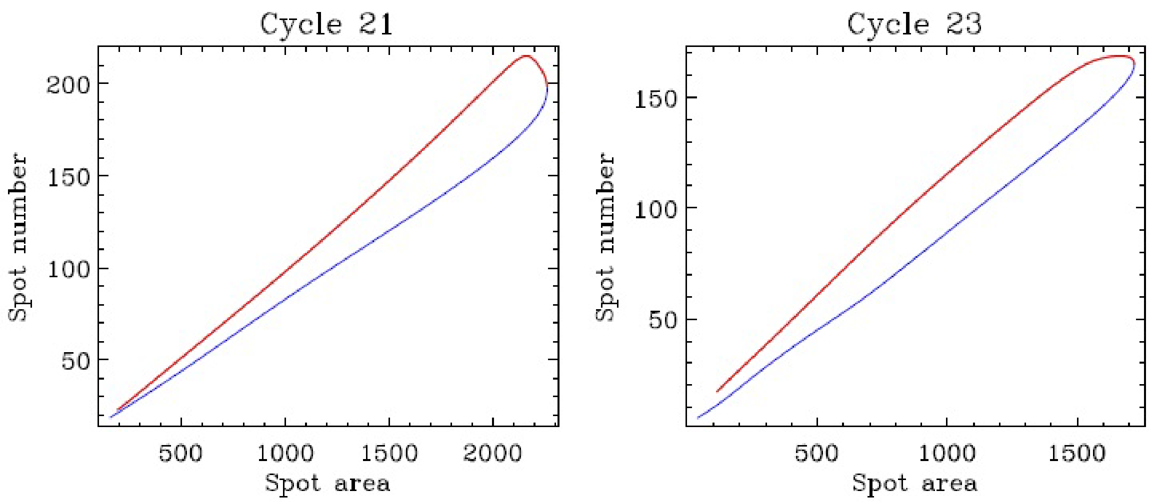

Figure 3. Trajectories of solar cycles 21 and 23 in the spot number — area coordinates. The growth and decline phases are indicated by red and blue lines respectively. The dates of minima and maxima refer to spot area

We find $P_{\text{false}} = 0.003$ for spot numbers and $P_{\text{false}} = 0.315$ for spot areas. The fact that the Waldmeier law is valid with 0.997 probability for spot numbers and absent with 32 % probability in spot areas adds little to understanding solar activity. It is desirable to find out the reason for such a difference.

The Wolf numbers are a combination of numbers of spots with numbers of spot groups (see Vitinsky et al. [1986] or Hathaway [2015] for more details), which does not generally account for spots' size. However, Nagovitsyn and Pevtsov [2016] and Nagovitsyn et al. [2016] discovered two populations of spots that differ by their characteristic size and lifetime. The numbers of relative small short-living and large long-living spots anticorrelate [Nagovitsyn et al., 2012]. These findings lead to the following explanation for the difference in the Waldmaier law between spot numbers and areas seen in Figure 2. At the beginning of a solar cycle, the contribution of small spots is relatively large and the Wolf number grows faster than spot areas. As a cycle approaches its maximum, the contribution of large spots increases. Therefore, the Wolf number comes through its maximum value earlier than the area (magnetic flux) of the spots. Trajectories of cycles 21 and 23 on the number-area diagrams of Figure 3 give examples of such a behaviour (see also Badalyan and Obridko [2011]). The trajectories outline loops where the growth phase goes above the phase of decline. It is remarkable that the contribution of the large spot population remains relatively large in the decline phase, and the decline trajectory goes below the trajectory of growth. This may be related to the equatorial migration of sunspot activity: the instability of magnetic flux concentration for sunspot formation strengthens with decreasing latitude [Kitchatinov, 2020].

Such a behaviour is, nonetheless, typical of relative strong cycles only. For seven cycles 17−23 with higher than average amplitude among the cycles shown in Figure 1, the mean duration of the growth phase for areas (4.42 years) is considerably longer compared to the spot numbers (4.30 years). The mean durations of this phase for all fourteen cycles – 4.59 for areas and 4.60 for numbers – differ little. The loop-shaped trajectories in Figure 3 are, therefore, not a general rule. Examples of the most significant deviations from this shape are given in Figure 4.

A larger value of the spot number-to-area ratio in cycle's growth phase compared to the phase of decline is nevertheless present in sunspot statistics. The relation between spot numbers and areas is conventionally approximated by liner regression [Hathaway, 2015]. Figure 5 shows the linear regression lines separately for growth and decline phases as well as for common statistics from cycle 21 to cycle 25, for which a homogeneous area data series from the same source is available [Hathaway, 2015]. The growth phase line goes above the line of decline phase and the lines diverge. A similar plot but with a somewhat smaller divergence angle can be found for cycles 12–25.

The activity growth follows on average the upper line of Figure 5. A transition from upper to lower line occurs in the epochs of maxima. If the transition proceeds smoothly, as in Figure 3, the growth phase for spot areas lasts longer compared to spot numbers. This statistical regularity is present on average only, and it is more pronounced for strong cycles because of the line divergence in Figure 5.

CONCLUSIONS

The high negative correlation between growth phase duration and amplitude in the solar cycle for spot numbers is most likely a consequence of the Wolf number definition and does not provide comprehensive information about the physics of solar activity. A weaker pronounced Waldmeier law for spot area (magnetic flux) has a statistical significance level of ~ 0.7. The cycle amplitude dependence of growth phase duration indicates a presence of nonlinear effects in the solar dynamo mechanism. Weakness of the Waldmeier law for spot areas suggests a weak nonlinearity and hence a small 'supercriticality' of the solar dynamo. This is in agreement with inferences from the stellar rotation data for the Sun [Metcalfe et al., 2016]. From this viewpoint, a stronger Waldmeier effect can be expected for stars rotating faster than the Sun, which seems to be real [Grag et al., 2019].

The ratio of spot numbers to spot areas is on average higher in the cycles' growth phase compared to the decline phase (Figures 3 and 5). The spots are, therefore, larger in the decline phase. This fact might be significant for the still poorly understood physics of sunspot formation.

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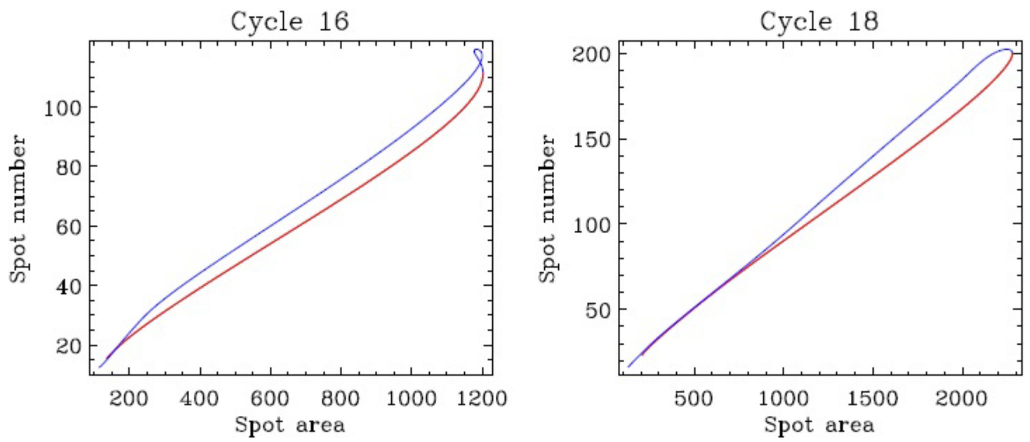

Figure 4. The same as in Figure 3, but for cycles 16 and 18 that show the most significant deviations from loop-shaped trajectories on the number-area plane.

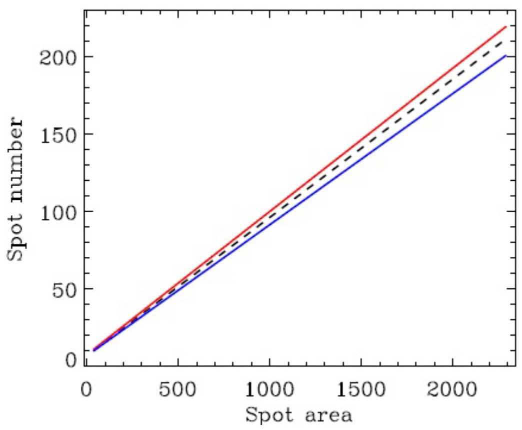

Figure 5. Lines of linear regression between spot numbers and areas for the phases of growth (red), decline (blue), and total statistics (dashed) of cycles 21–25

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