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DISTRIBUTION OF TEMPERATURE COEFFICIENT DENSITY FOR MUONS IN THE ATMOSPHERE

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Abstract. To date, several dozens of new muon detectors have been built. When studying cosmic-ray intensity variations with these detectors, located deep in the atmosphere, it is necessary to calculate all characteristics, including the distribution of temperature coefficient density for muons in the atmosphere, taking into account their specific geometry. For this purpose, we calculate the density of temperature coefficients of muon intensity in the atmosphere at various zenith angles of detection at sea level and at various depths underground for different absorption ranges of primary protons and pions in the atmosphere.

Keywords: cosmic rays, muons, temperature, atmosphere.

INTRODUCTION

When using data from muon telescopes in the analysis of cosmic ray variations, we should take into account the contribution of atmospheric effects, mainly pressure and temperature. The pressure effect of muon intensity is estimated quite simply as being determined by one parameter, pressure at the observation level, but the temperature effect is not so unambiguous. The temperature effect of muons is determined by several parameters characterizing atmospheric conditions from the generation layer to the level of muon detection. The empirical method of determining the temperature effect [Duperier, 1949] usually involves using two parameters (for example, the height and temperature of the muon generation layer), which are found from the correlation with the observed intensity. This method is simple and free from possible errors in theoretical calculations of the coefficients. However, the empirical method ignores the mass distribution that is completely and unambiguously controlled by the temperature section from the observation level to the atmosphere boundary. The integral method of taking into account the temperature effect [Dorman, 1957] involves using a complete temperature section of the atmosphere and is free from the above shortcomings. The integral method requires regular data on aerological sounding and distribution of temperature coefficient density for atmospheric muons. The purpose of this paper is to find the distribution function of temperature coefficient density *W*(*h*) for the muon telescope of the cosmic ray station Novosibirsk and the underground complex of muon detectors in Yakutsk located at a depth of 0, 7, 20, and 40 m of water equivalent (w.e.).

CALCULATING THE DISTRIBUTION OF TEMPERATURE COEFFICIENT DENSITY FOR ATMOSPHERIC MUONS

The function *W*(*h*) representing the temperature coefficient density, was first introduced in 1956 [Dorman, Feinberg, 1956]. This function has been theoretically calculated for various conditions of muon detection [Dorman, 1957; Kuzmin, 1964; Dorman, Janke, 1971; Dmitrieva et al., 2009; Berkova et al., 2008; Volkova, 2013]. Methods of calculating *W*(*h*) in these papers differ slightly and are based on ideas of the work [Dorman, 1957]. However, results of these studies do not fully agree since the calculations were made for different values of initial parameters. Basing on the results obtained in [Dorman, 1957; Dorman, Janke, 1971], we represent the directional muon intensity as

$$N_\mu\left(\Delta\varepsilon, h_0, \theta\right) = \int_0^{h_0} dh_2 \int_0^{h_2} dh_1 \int_{\varepsilon_{\min}}^{\infty} d\varepsilon_\mu \int_{\varepsilon_{\pi^+}}^{\varepsilon_{\pi^-}} d\varepsilon_\pi F\left(\varepsilon_\pi, \varepsilon_\mu, h_1, h_2, h_0, \theta\right), \tag{1}$$

where

$$F\left(\varepsilon_\pi, \varepsilon_\mu, h_0, h_1, h_2, \theta\right) = \frac{m_\pi c}{\tau_\pi}\frac{f_\pi\left(\varepsilon_\pi, h_1, \theta\right)}{\varepsilon_\pi \cos\theta} \times \frac{1}{\left(\varepsilon_{\pi^-} - \varepsilon_{\pi^+}\right)\rho\left(h_2\right)} \exp\left(-\frac{h_2 - h_1}{l_\pi \cos\theta}\right) \times \exp\left(-\frac{m_\pi c}{\tau_\pi \varepsilon_\pi \cos\theta}\int_{h_1}^{h_2}\frac{dh}{\rho\left(h\right)}\right) \times \exp\left(-\frac{m_\mu c}{\tau_\mu}\int_{h_2}^{h_0}\frac{dh}{\rho\left(h\right)\left[\varepsilon_\mu \cos\theta - \alpha_\mu\left(h - h_2\right)\right]}\right). \tag{2}$$

Here $\varepsilon_{\pi^+} \approx \varepsilon_\mu$, $\varepsilon_{\pi^-} = \frac{\varepsilon_\mu}{\alpha^2}$, $\varepsilon_{\min} = \frac{a_\mu(h_0 - h_2) + \Delta\varepsilon}{\alpha\cos\theta}$ are the minimum energy of pions, muons from which can be recorded by the detector, $\Delta\varepsilon$ is the minimum energy of muons recorded by the detector, ε_μ is the total muon energy, *h* is the atmospheric depth, h_0 is the observation level, h_1 is the pion generation level, h_2 is the muon generation level, θ is the zenith angle of arrival; m_π and m_μ are the pion and muon masses at rest, τ_π and τ_μ are the lifetimes of pion and muon at rest respectively;

$\rho(h)=gh/RT(h)$ is the specific weight of air at a height with pressure h; g is the gravitational acceleration, R is the specific gas constant, $T(h)$ is the air temperature in Kelvin at a height with pressure h; c is the speed of light, $\alpha=m_\mu/m_\pi$, a_μ is the energy loss of particles for ionization, l_π is the pion range before nuclear capture, $f_\pi(\varepsilon_\pi,h_1,\theta)=\frac{A}{\varepsilon_\pi^\gamma}\exp\left(-\frac{h_1}{L\cos\theta}\right)$ is the differential pion spectrum, A is the constant, L is the average absorption range for the nucleon component; γ is the exponent of the pion differential spectrum. Muon intensity variations under a temperature distribution variation (temperature effect) are found by varying expression (1) with respect to the corresponding independent variable:

$$\delta N_\mu(\Delta\varepsilon,h_0,\theta)= =\int_0^{h_0}dh_2\int_0^{h_2}dh_1\int_{\varepsilon_{мин}}^{\infty}d\varepsilon_\mu\int_{\varepsilon_\pi+}^{\varepsilon_\pi-}d\varepsilon_\pi F(\varepsilon_\pi,\varepsilon_\mu,h_1,h_2,h_0,\theta)- -\int_0^{h_0}dh_2\int_0^{h_2}dh_1\int_{\varepsilon_{min}}^{\infty}d\varepsilon_\mu\int_{\varepsilon_\pi+}^{\varepsilon_\pi-}d\varepsilon_\pi F(\varepsilon_\pi,\varepsilon_\mu,h_1,h_2,h_0,\theta)\times \times\frac{b_\pi}{\varepsilon_\pi\cos\theta}\int_{h_1}^{h_2}\frac{dh}{h}\delta T(h)- -\int_0^{h_0}dh_2\int_0^{h_2}dh_1\int_{\varepsilon_{min}}^{\infty}d\varepsilon_\mu\int_{\varepsilon_\pi+}^{\varepsilon_\pi-}d\varepsilon_\pi F(\varepsilon_\pi,\varepsilon_\mu,h_1,h_2,h_0,\theta)b_\mu\times \times\int_{h_2}^{h_0}\frac{\delta T(h)dh}{h[\varepsilon_\mu\cos\theta-a_\mu(h-h_2)]}, \quad (3)$$

where $b_\pi=\frac{m_\pi cR}{\tau_\pi g}$, $b_\mu=\frac{m_\mu cR}{\tau_\mu g}$. Resting on (3), we can write the relative muon intensity variation, caused by the temperature effect, as follows:

$$\frac{\delta N(\Delta\varepsilon,h_0,\theta)}{N(\Delta\varepsilon,h_0,\theta)}=\int_0^{h_0}W_T(\Delta\varepsilon,h_0,\theta)\delta T(h)dh. \quad (4)$$

The temperature coefficient density function

$$W_T(\Delta\varepsilon,h,h_0,\theta)=W_T^\mu(\Delta\varepsilon,h,h_0,\theta)+W_T^\pi(\Delta\varepsilon,h,h_0,\theta) \quad (5)$$

includes the muon temperature effect

$$W_T^\mu(\Delta\varepsilon,h,h_0,\theta)= =-\frac{1}{N_\mu}\int_0^{h_0}dh_2\int_0^{h_2}dh_1\int_{\varepsilon_{min}}^{\infty}d\varepsilon_\mu\times \times\int_{\varepsilon_\pi+}^{\varepsilon_\pi-}d\varepsilon_\pi\frac{b_\mu F(\varepsilon_\pi,\varepsilon_\mu,h_1,h_2,h_0,\theta)}{h[\varepsilon_\mu\cos\theta-a_\mu(h-h_2)]}, \quad (6)$$

caused by muon decay and ionization losses in the atmosphere (muon effect), and the temperature effect of pions

$$W_T^\pi(\Delta\varepsilon,h,h_0,\theta)= =\frac{1}{N_\mu}\int_0^{h}dh_1\int_{\varepsilon_{мин}}^{\infty}d\varepsilon_\mu\int_{\varepsilon_\pi+}^{\varepsilon_\pi-}d\varepsilon_\pi\frac{1}{T(h)}F(\varepsilon_\pi,\varepsilon_\mu,h_1,h_2,h_0,\theta)- -\frac{1}{N_\mu}\int_0^{h}dh_2\int_0^{h_2}dh_1\int_{\varepsilon_{min}}^{\infty}d\varepsilon_\mu\int_{\varepsilon_\pi+}^{\varepsilon_\pi-}d\varepsilon_\pi\frac{b_\pi F(\varepsilon_\pi,\varepsilon_\mu,h_1,h,h_0,\theta)}{h\varepsilon_\pi\cos\theta}, \quad (7)$$

caused by pion decay and capture (pion effect).

CALCULATION RESULTS

The function $W(h)$ is calculated for the direction of arrival of particles from $\theta=0°$ and for $\gamma=2.5$ (Figure 1). By comparison, Figure 1 presents the results obtained by other authors (also for $\theta=0°$ and $\gamma=2.5$–2.8). Table 1 lists the initial parameters used in the calculations.

To compare our calculation results with the results received by Dorman and Yanke [1971], we select close values of the initial parameters (see Table 1). Nonetheless, there are small discrepancies (Figure 1). They arise for the following reasons. Dorman and Yanke [1971] use a series of approximations with integration over h_1 and ε_{min}, namely, they apply the mean value theorem for intermediate integrals with allowance for slowly varying integrands, use estimates, and calculate integrals with the method of successive approximations, whereas we make a direct calculation.

There are also discrepancies between vertical profiles of atmospheric temperature for Moscow and Novosibirsk. It should be concluded that the results presented in Figure 1 are difficult to compare (except for the results of this paper and [Dorman, Yanke, 1971]). This discrepancy is primarily attributed to tangible differences in values of the initial parameters used in calculations (Table 1). We use the standard atmosphere model, consider muon losses as constant, discuss only the contribution of pions since the instrument in Novosibirsk can detect muons up to 200 GeV (other muon production channels are activated only after 10^4 GeV [Kochanov, 2008]).

DEPENDENCE OF *W*(*h*) DISTRIBUTION ON PROTON AND PION ABSORPTION RANGES IN THE ATMOSPHERE

The above results were obtained for proton absorption ranges from 75 g/cm^2 [Kuzmin 1964] to 120 g/cm^2 [Dorman, Yanke 1971] and pion absorption ranges from 60 g/cm^2 [Dorman, Yanke 1971] to 120 g/cm^2 [Dmitrieva et al., 2009]. At the same time, different authors take the spectrum index γ equal to 2.5; 2.7; 2.8. Hereafter, in calculations, taking into account the latest data [Murzin, 2017; Karelin et al., 2011], we set γ equal to 2.75 [Karelin et al., 2011]. The $W(h)$ function for different values of proton and pion absorption ranges is computed for the vertical muon intensity at sea level ($\theta=0$) at $\Delta\varepsilon=0.6$ GeV. The calculation results thus obtained are shown in Figure 2.

There is a marked dependence of $W(h)$ distribution on absorption ranges, especially on the absorption range of primary protons in the atmosphere (Figure 2, *a*).

DISTRIBUTIONS OF *W*(*h*) FOR INTENSITY OF MUONS, DETECTED AT VARIOUS ZENITH ANGLES

The calculation is made for different zenith angles θ of the telescope, to each of which corresponds the effective threshold $\Delta\varepsilon$ of muon detection (Table 2).

In all the calculations, we take the standard atmosphere, whose temperature distribution $T(h)$ is shown in Figure 3.

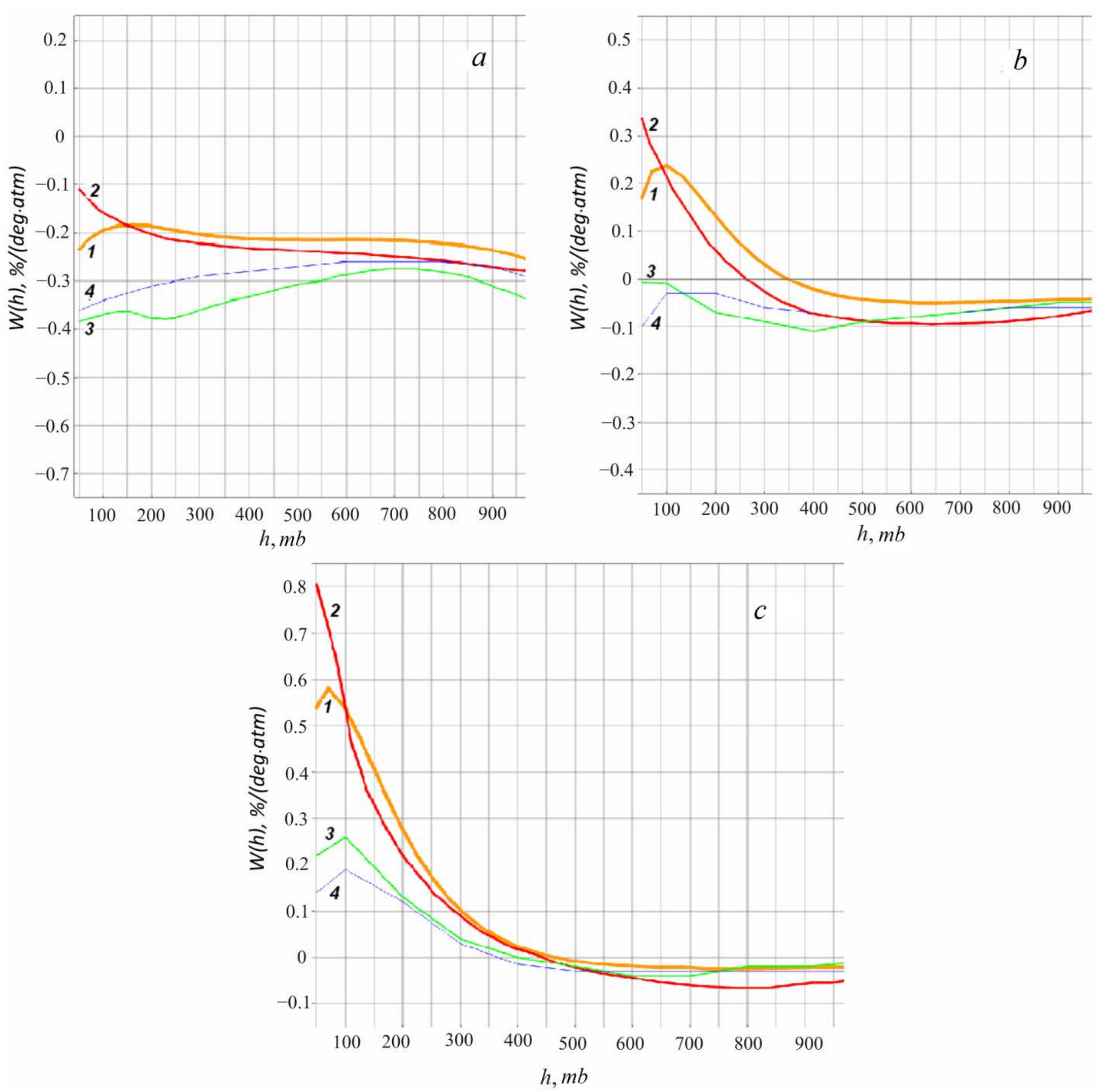

Figure 1. Distributions of density of temperature coefficients of muon intensity in the atmosphere, obtained by different authors (1 – this paper, 2 – [Dorman, Yanke, 1971], 3 – [Kuzmin, 1964], 4 – [Dmitrieva et al., 2009]) for different values of minimum energy of muons Δε detected: *a* – Δε=0.24–0.5 GeV; *b* – Δε=2.2–6.4 GeV; *c* – Δε=10.0–16.2 GeV

Table 1

Initial parameters used in calculations

Author	Figure 1, *a*				Figure 1, *b*				Figure 1, *c*			
	Δε, GeV	γ	*l*, g/cm^2	*L*, g/cm^2	Δε, GeV	γ	*l*, g/cm^2	*L*, g/cm^2	Δε, GeV	γ	*l*, g/cm^2	*L*, g/cm^2
1	0.4	2.5	60	120	6.4	2.5	60	120	14.4	2.5	60	120
2	0.4	2.5	60	120	6.4	2.5	60	120	14.4	2.5	60	120
3	0.24	2.8	75	75	4.5	2.8	75	75	16.2	2.8	75	75
4	0.5	2.7	120	110	2.2	2.7	120	110	10.0	2.7	120	110

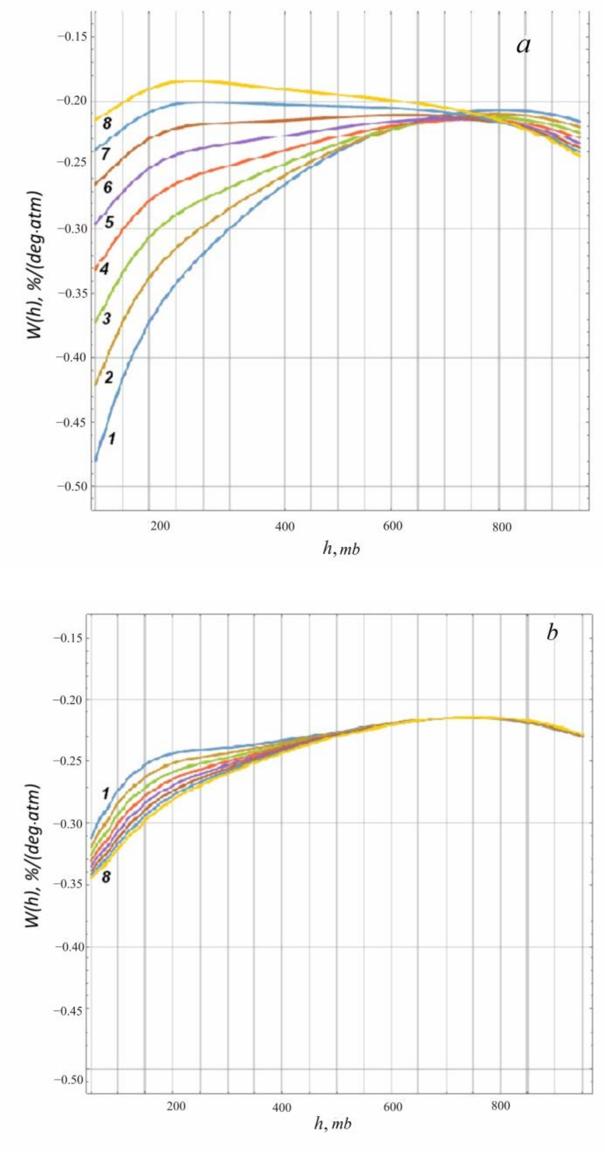

Figure 2. Distributions of $W(h)$ at constant values of the pion absorption range l=110 g/cm^2 (*a*) for proton absorption ranges L from 80 to 150 g/cm^2 (curves 1–8 respectively); as well as the proton absorption range L=110 g/cm^2 (*b*) for pion absorption ranges l from 80 to 150 g/cm^2 (curves 1–8 respectively)

Table 2

Parameters of the muon telescope in Novosibirsk

θ, °	0	30	40	50	60	67	71
Δε, GeV	0.6	0.69	0.78	0.93	1.2	1.5	1.8

We set the proton absorption range in the energy range 5–200 GeV, according to [Murzin, Sarycheva, 1968; Hayakawa, 1973; Murzin, 2007; Sarycheva, 2007] equal to 110 g/cm^2. The calculation is carried out for various pion absorption ranges l from 70 to 110 g/cm^2. The calculation results are shown in Figure 4 for various zenith angles of muon detection.

More significant changes in the $W(h)$ distribution as a function of l occur with increasing zenith angle θ of muon detection.

W(*h*) DISTRIBUTION FOR MUONS DETECTED UNDERGROUND

The calculation is made for zenith angles of muon detection of 0, 30, and 60° at depths of 0, 7, 20, and 40 m. We use the following values of initial parameters: γ=2.75; L=110 g/cm^2; l=120 g/cm^2. In this case, Δε takes the following values: 0.24 GeV (sea level); 1.6, 1.85, 3.2 GeV (7 m w.e.); 4.5, 5.2, 9.0 GeV (20 m w.e.); 9.5, 10.97, 19.0 GeV (40 m w.e.). The results are presented in Figure 5.

The results of calculation of the temperature coefficient density for atmospheric muons, detected with the instruments in Novosibirsk and Yakutsk, are presented in digital form in Tables 3–8 (see Appendix).

CONCLUSION

By calculations, we have found distributions of the density of temperature coefficients for atmospheric muons $W(h)$ for various zenith angles of detection at sea level and at various depths below the ground. Noteworthy is the observed dependence of $W(h)$ on proton and pion absorption ranges in the atmosphere, especially on their ratio ($L \leq l$ or $L \geq l$). While the proton absorption range in the upper atmospheric layers is known, the pion absorption range is not.

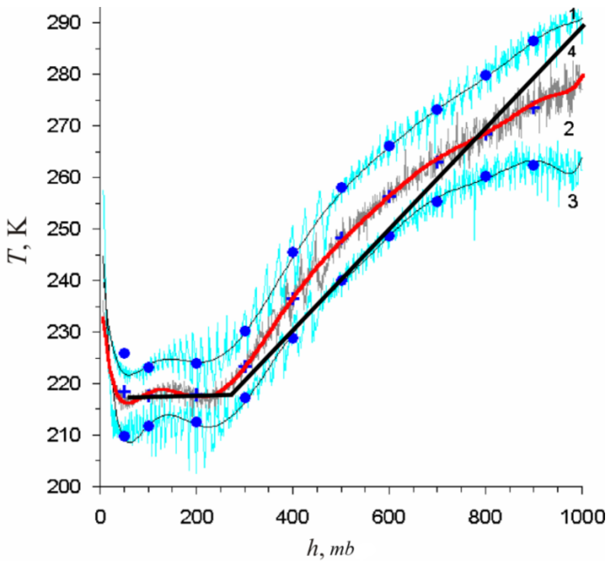

Figure 3. Altitudinal variations of atmospheric temperature over Novosibirsk: 1 – summer, 2 – spring and autumn, 3 – winter, 4 – piecewise linear approximation function

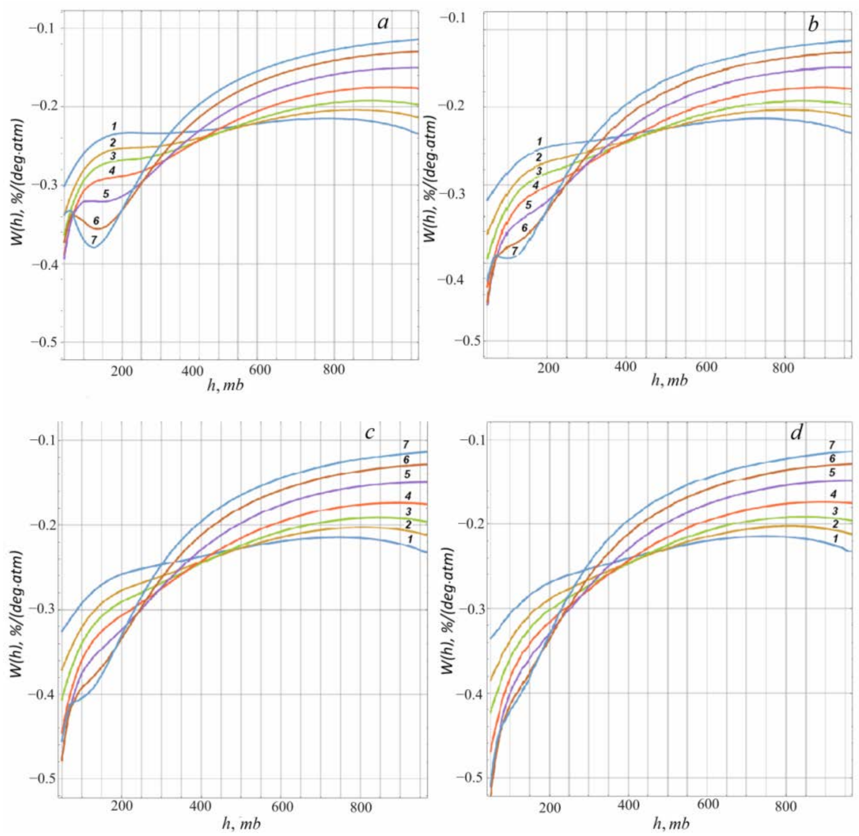

Figure 4. Distributions of $W(h)$ for zenith angles of muon detection from 0 to 71° (curves 1–7 respectively) for pion absorption ranges of 70 (*a*), 90 (*b*), 100 (*c*), 110 g/cm^2 (*d*)

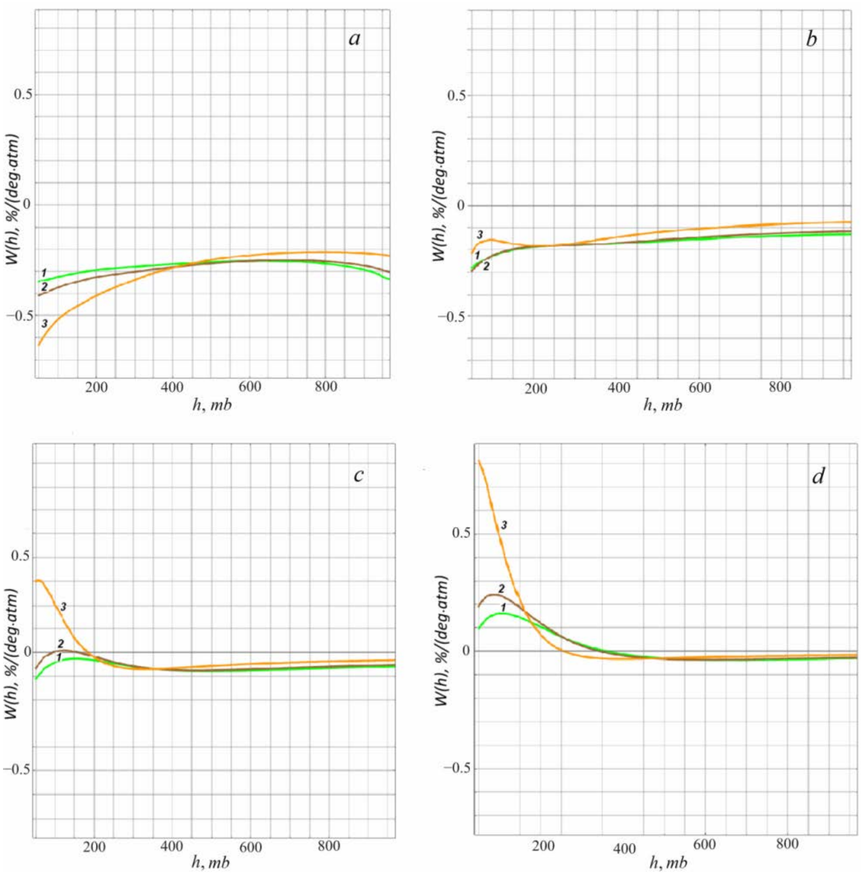

Figure 5. Distributions of $W(h)$ for muons detected at sea level (*a*) and underground at depths of 7 (*b*), 20 (*c*), and 40 m (*d*) at zenith angles of 0, 30, and 60° (curves 1–3 respectively)

The comparison shows that the $W(h)$ distributions obtained in different papers are difficult to compare (except for the results of this paper and [Dorman, Janke, 1971]). The discrepancy between the results is primarily attributed to the tangible differences between initial parameters used by the authors for calculations.

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APPENDIX

Table 3

Density of temperature coefficients for atmospheric muons detected at sea level (Novosibirsk) at different zenith angles, with absorption ranges of protons L=110 g/cm^2 and pions l=70 g/cm^2

Atmospheric depth, *h*, mbar	Zenith angle, θ, °						
	0	30	40	50	60	67	71
40	–0.31212	–0.35456	–0.3856	–0.41826	–0.43064	–0.40305	–0.34815
70	–0.28198	–0.30946	–0.32671	–0.34089	–0.3408	–0.33636	–0.33344
100	–0.25977	–0.28091	–0.29424	–0.30746	–0.32065	–0.34557	–0.36922
130	–0.24593	–0.26516	–0.27856	–0.29539	–0.32023	–0.35538	–0.37906
160	–0.23807	–0.25716	–0.27158	–0.29139	–0.3205	–0.35189	–0.3654
190	–0.23408	–0.25339	–0.26846	–0.28914	–0.31633	–0.33791	–0.34047
220	–0.23317	–0.25241	–0.26728	–0.2867	–0.30808	–0.3187	–0.31276
250	–0.23351	–0.25204	–0.26586	–0.28246	–0.29653	–0.29768	–0.28626
280	–0.23383	–0.25105	–0.26318	–0.27619	–0.28305	–0.27709	–0.2626
310	–0.23379	–0.24925	–0.25932	–0.26849	–0.26901	–0.25811	–0.24214
340	–0.23329	–0.24667	–0.25453	–0.25998	–0.25529	–0.24117	–0.22467
370	–0.23232	–0.24348	–0.24911	–0.25118	–0.24238	–0.22628	–0.20978
400	–0.23096	–0.23984	–0.24336	–0.24244	–0.23052	–0.21329	–0.19705
430	–0.22929	–0.23593	–0.23749	–0.23404	–0.21979	–0.20199	–0.18611
460	–0.22742	–0.23192	–0.2317	–0.22612	–0.21018	–0.19213	–0.17665
490	–0.22545	–0.22792	–0.22612	–0.21877	–0.20162	–0.18353	–0.16841
520	–0.22346	–0.22405	–0.22086	–0.21205	–0.19403	–0.17599	–0.1612
550	–0.22154	–0.22038	–0.21597	–0.20597	–0.18731	–0.16937	–0.15486
580	–0.21976	–0.21699	–0.2115	–0.20051	–0.18137	–0.16353	–0.14926
610	–0.21818	–0.21392	–0.20748	–0.19565	–0.17613	–0.15838	–0.14429
640	–0.21686	–0.21122	–0.20393	–0.19138	–0.17153	–0.15383	–0.13988
670	–0.21584	–0.20891	–0.20085	–0.18765	–0.16749	–0.14981	–0.13595
700	–0.21518	–0.20702	–0.19824	–0.18445	–0.16397	–0.14625	–0.13245
730	–0.21492	–0.20557	–0.19611	–0.18175	–0.16092	–0.14311	–0.12933
760	–0.2151	–0.20459	–0.19447	–0.17953	–0.15831	–0.14036	–0.12655
790	–0.21578	–0.2041	–0.19331	–0.17777	–0.15609	–0.13795	–0.12408
820	–0.21701	–0.20413	–0.19264	–0.17647	–0.15426	–0.13586	–0.12189
850	–0.21887	–0.20471	–0.1925	–0.17562	–0.15279	–0.13407	–0.11996
880	–0.22143	–0.20591	–0.19288	–0.17522	–0.15166	–0.13257	–0.11828
910	–0.22481	–0.20776	–0.19385	–0.17528	–0.15088	–0.13133	–0.11681
940	–0.2291	–0.21036	–0.19542	–0.17582	–0.15043	–0.13035	–0.11556
970	–0.23422	–0.21372	–0.19766	–0.17687	–0.15033	–0.12962	–0.11452
1000	–0.23127	–0.21593	–0.20018	–0.17843	–0.15058	–0.12914	–0.11367

Table 4

Density of temperature coefficients for atmospheric muons detected at sea level (Novosibirsk) at various zenith angles, with absorption ranges of protons L=110 g/cm^2 and pions l=90 g/cm^2

Atmospheric depth, *h*, mbar	Zenith angle, θ, °						
	0	30	40	50	60	67	71
40	–0.32714	–0.37589	–0.41366	–0.45824	–0.49223	–0.48824	–0.44973
70	–0.30395	–0.33806	–0.36149	–0.38476	–0.3959	–0.39583	–0.38968
100	–0.28374	–0.31005	–0.32745	–0.34513	–0.35983	–0.37867	–0.39365
130	–0.26921	–0.29173	–0.30704	–0.32455	–0.34532	–0.37162	–0.38783
160	–0.25933	–0.27999	–0.29462	–0.31267	–0.3355	–0.35893	–0.36753
190	–0.25278	–0.27229	–0.28641	–0.30406	–0.32476	–0.34034	–0.34009
220	–0.24904	–0.2675	–0.28077	–0.29673	–0.31241	–0.31894	–0.31157
250	–0.24666	–0.26381	–0.27572	–0.28895	–0.29844	–0.29699	–0.28489
280	–0.24458	–0.26009	–0.27025	–0.28021	–0.28358	–0.27603	–0.26127
310	–0.2425	–0.25609	–0.26428	–0.27084	–0.26878	–0.25695	–0.2409
340	–0.24028	–0.25178	–0.25792	–0.2612	–0.25465	–0.24	–0.22352
370	–0.23788	–0.24723	–0.25135	–0.25164	–0.24153	–0.22515	–0.20871
400	–0.23534	–0.24254	–0.24473	–0.24241	–0.22957	–0.21221	–0.19604
430	–0.23271	–0.23781	–0.23824	–0.23368	–0.21881	–0.20096	–0.18515

460	–0.23005	–0.23316	–0.23199	–0.22554	–0.20919	–0.19115	–0.17573
490	–0.22744	–0.22867	–0.22608	–0.21806	–0.20064	–0.18258	–0.16753
520	–0.22492	–0.22442	–0.22057	–0.21125	–0.19306	–0.17508	–0.16035
550	–0.22258	–0.22046	–0.2155	–0.20511	–0.18635	–0.16848	–0.15404
580	–0.22045	–0.21684	–0.2109	–0.19961	–0.18043	–0.16267	–0.14846
610	–0.21858	–0.21359	–0.20679	–0.19474	–0.17522	–0.15754	–0.14352
640	–0.21702	–0.21075	–0.20316	–0.19045	–0.17063	–0.15301	–0.13913
670	–0.21581	–0.20833	–0.20002	–0.18672	–0.1666	–0.149	–0.13521
700	–0.21498	–0.20635	–0.19737	–0.18351	–0.16309	–0.14545	–0.13173
730	–0.21457	–0.20482	–0.19521	–0.18081	–0.16005	–0.14233	–0.12862
760	–0.21464	–0.20378	–0.19354	–0.17858	–0.15744	–0.13958	–0.12585
790	–0.21521	–0.20323	–0.19235	–0.17682	–0.15523	–0.13718	–0.12339
820	–0.21635	–0.20321	–0.19166	–0.17551	–0.1534	–0.1351	–0.1212
850	–0.21812	–0.20375	–0.19149	–0.17465	–0.15192	–0.13331	–0.11928
880	–0.2206	–0.2049	–0.19185	–0.17423	–0.1508	–0.13181	–0.1176
910	–0.22389	–0.20671	–0.19279	–0.17428	–0.15001	–0.13057	–0.11614
940	–0.22809	–0.20925	–0.19433	–0.17481	–0.14956	–0.12959	–0.11489
970	–0.23306	–0.21254	–0.19653	–0.17583	–0.14945	–0.12885	–0.11384
1000	–0.2298	–0.21461	–0.19898	–0.17736	–0.14968	–0.12837	–0.11299

Table 5

Density of temperature coefficients for atmospheric muons detected at sea level (Novosibirsk) at various zenith angles, with absorption ranges of protons *L*=110 g/cm^2 and pions *l*=100 g/cm^2

Atmospheric depth, *h*, mbar	Zenith angle, θ, °						
	0	30	40	50	60	67	71
40	–0.33243	–0.38345	–0.42367	–0.47265	–0.51477	–0.51998	–0.48822
70	–0.3119	–0.34848	–0.37426	–0.40107	–0.41682	–0.41902	–0.41219
100	–0.29257	–0.32088	–0.33991	–0.35948	–0.37517	–0.39209	–0.40395
130	–0.27792	–0.30177	–0.31792	–0.33589	–0.35541	–0.37849	–0.39178
160	–0.26738	–0.28874	–0.30356	–0.3211	–0.3417	–0.36205	–0.36864
190	–0.25993	–0.27962	–0.29347	–0.31008	–0.32835	–0.34151	–0.34008
220	–0.25516	–0.27342	–0.28614	–0.30083	–0.31432	–0.31917	–0.31115
250	–0.25177	–0.26846	–0.27969	–0.29164	–0.29932	–0.29678	–0.28438
280	–0.24879	–0.26369	–0.27313	–0.28192	–0.28388	–0.27565	–0.26075
310	–0.24593	–0.25884	–0.26632	–0.27186	–0.26875	–0.2565	–0.24041
340	–0.24304	–0.25385	–0.25933	–0.26175	–0.25443	–0.23955	–0.22306
370	–0.24009	–0.24876	–0.25228	–0.25188	–0.24121	–0.22471	–0.20828
400	–0.23708	–0.24364	–0.24532	–0.24244	–0.22921	–0.21179	–0.19564
430	–0.23407	–0.23859	–0.23857	–0.23356	–0.21842	–0.20055	–0.18477
460	–0.2311	–0.23368	–0.23214	–0.22533	–0.2088	–0.19076	–0.17537
490	–0.22823	–0.22899	–0.22609	–0.21779	–0.20025	–0.18221	–0.16718
520	–0.22551	–0.22458	–0.22047	–0.21094	–0.19268	–0.17471	–0.16002
550	–0.22299	–0.22051	–0.21533	–0.20478	–0.18598	–0.16813	–0.15372
580	–0.22072	–0.21679	–0.21068	–0.19927	–0.18007	–0.16233	–0.14815
610	–0.21874	–0.21347	–0.20652	–0.19438	–0.17486	–0.15721	–0.14321
640	–0.21708	–0.21057	–0.20286	–0.19009	–0.17027	–0.15268	–0.13883
670	–0.21579	–0.2081	–0.1997	–0.18635	–0.16625	–0.14868	–0.13492
700	–0.2149	–0.20609	–0.19704	–0.18314	–0.16275	–0.14514	–0.13144
730	–0.21444	–0.20453	–0.19486	–0.18044	–0.15971	–0.14202	–0.12834
760	–0.21445	–0.20346	–0.19317	–0.17821	–0.1571	–0.13927	–0.12557
790	–0.21499	–0.20289	–0.19198	–0.17644	–0.15489	–0.13688	–0.12311
820	–0.21609	–0.20285	–0.19128	–0.17513	–0.15306	–0.13479	–0.12093
850	–0.21782	–0.20338	–0.1911	–0.17427	–0.15159	–0.13301	–0.11901
880	–0.22027	–0.20451	–0.19145	–0.17385	–0.15046	–0.13151	–0.11733
910	–0.22353	–0.2063	–0.19237	–0.17389	–0.14967	–0.13027	–0.11587
940	–0.22769	–0.20882	–0.19391	–0.17441	–0.14921	–0.12928	–0.11462
970	–0.23261	–0.21208	–0.19609	–0.17543	–0.1491	–0.12855	–0.11358
1000	–0.22923	–0.2141	–0.19852	–0.17695	–0.14933	–0.12806	–0.11272

Table 6

Density of temperature coefficients for atmospheric muons detected at sea level (Novosibirsk) at various zenith angles, with absorption ranges of protons L=110 g/cm^2 and pions l=110 g/cm^2

Atmospheric depth, *h*, mbar	Zenith angle, θ, °						
	0	30	40	50	60	67	71
40	–0.33678	–0.38968	–0.43195	–0.48461	–0.53363	–0.5468	–0.52101
70	–0.31852	–0.35719	–0.38497	–0.41483	–0.43467	–0.43907	–0.43191
100	–0.29999	–0.33003	–0.35047	–0.37174	–0.38845	–0.40394	–0.41322
130	–0.28528	–0.31031	–0.32723	–0.34568	–0.36428	–0.38467	–0.39545
160	–0.27424	–0.29623	–0.31126	–0.32845	–0.34722	–0.36493	–0.36975
190	–0.26606	–0.28594	–0.2996	–0.31537	–0.33158	–0.34264	–0.34013
220	–0.26043	–0.27855	–0.29083	–0.30447	–0.31607	–0.31942	–0.31082
250	–0.25618	–0.27251	–0.28317	–0.29405	–0.30016	–0.29662	–0.28393
280	–0.25244	–0.26684	–0.27566	–0.28345	–0.28418	–0.27533	–0.2603
310	–0.2489	–0.26125	–0.26813	–0.27278	–0.26874	–0.25612	–0.23998
340	–0.24544	–0.25566	–0.26058	–0.26226	–0.25425	–0.23916	–0.22266
370	–0.24201	–0.2501	–0.25312	–0.25211	–0.24095	–0.22433	–0.2079
400	–0.23861	–0.24462	–0.24585	–0.24247	–0.2289	–0.21142	–0.19529
430	–0.23527	–0.23927	–0.23888	–0.23347	–0.21809	–0.20019	–0.18444
460	–0.23202	–0.23414	–0.23227	–0.22516	–0.20846	–0.19042	–0.17505
490	–0.22893	–0.22928	–0.2261	–0.21756	–0.19992	–0.18188	–0.16688
520	–0.22603	–0.22474	–0.22039	–0.21068	–0.19235	–0.1744	–0.15973
550	–0.22336	–0.22055	–0.21519	–0.20449	–0.18565	–0.16782	–0.15343
580	–0.22096	–0.21676	–0.21048	–0.19897	–0.17975	–0.16203	–0.14788
610	–0.21888	–0.21337	–0.20629	–0.19407	–0.17454	–0.15692	–0.14295
640	–0.21714	–0.21042	–0.20261	–0.18978	–0.16996	–0.1524	–0.13857
670	–0.21578	–0.20791	–0.19943	–0.18604	–0.16595	–0.1484	–0.13467
700	–0.21483	–0.20586	–0.19675	–0.18282	–0.16245	–0.14486	–0.13119
730	–0.21432	–0.20428	–0.19456	–0.18012	–0.15941	–0.14175	–0.12809
760	–0.2143	–0.20319	–0.19286	–0.17789	–0.1568	–0.13901	–0.12533
790	–0.21479	–0.2026	–0.19165	–0.17612	–0.1546	–0.13661	–0.12287
820	–0.21586	–0.20254	–0.19095	–0.1748	–0.15277	–0.13453	–0.12069
850	–0.21757	–0.20305	–0.19076	–0.17394	–0.15129	–0.13275	–0.11878
880	–0.21999	–0.20416	–0.1911	–0.17352	–0.15016	–0.13125	–0.11709
910	–0.22321	–0.20594	–0.19201	–0.17355	–0.14937	–0.13001	–0.11564
940	–0.22734	–0.20844	–0.19354	–0.17407	–0.14892	–0.12902	–0.11439
970	–0.23221	–0.21168	–0.1957	–0.17508	–0.1488	–0.12829	–0.11334
1000	–0.22874	–0.21366	–0.19811	–0.17659	–0.14903	–0.1278	–0.11249

Table 7

Density of temperature coefficients for atmospheric muons detected at sea level and below the ground at a depth of 7 m (Yakutsk) at zenith angles of 0, 30, and 60°, with absorption ranges of protons L=110 g/cm^2 and pions l=120 g/cm^2

Atmospheric depth, *h*, mbar	0 m w.e.			Atmospheric depth, *h*, mbar	7 m w.e.		
	0	30	60		0	30	60
40	–0.3463	–0.41457	–0.66969	40	–0.29103	–0.31187	–0.25137
70	–0.33797	–0.39425	–0.57545	70	–0.25586	–0.26084	–0.16657
100	–0.32506	–0.37255	–0.51635	100	–0.22894	–0.22732	–0.15504
130	–0.31334	–0.3547	–0.47643	130	–0.21046	–0.20715	–0.16491
160	–0.3037	–0.34059	–0.44532	160	–0.19827	–0.19547	–0.17607
190	–0.296	–0.32932	–0.41834	190	–0.19043	–0.18886	–0.18253
220	–0.29027	–0.32052	–0.39403	220	–0.18592	–0.18552	–0.18394
250	–0.28568	–0.31302	–0.37139	250	–0.18311	–0.18343	–0.18102
280	–0.28157	–0.30603	–0.35027	280	–0.18085	–0.18136	–0.17512
310	–0.27773	–0.29935	–0.33087	310	–0.17871	–0.17895	–0.16762
340	–0.27409	–0.29293	–0.31331	340	–0.17646	–0.17609	–0.15949
370	–0.27062	–0.28676	–0.2976	370	–0.17404	–0.17282	–0.15135
400	–0.26733	–0.2809	–0.28368	400	–0.17143	–0.16923	–0.14355
430	–0.26425	–0.2754	–0.27143	430	–0.16865	–0.16542	–0.13627

460	–0.26144	–0.27031	–0.26071	460	–0.16576	–0.16149	–0.12958
490	–0.25893	–0.26567	–0.25138	490	–0.1628	–0.15754	–0.1235
520	–0.25678	–0.26154	–0.2433	520	–0.15982	–0.15363	–0.11799
550	–0.25504	–0.25796	–0.23635	550	–0.15686	–0.14983	–0.11301
580	–0.25376	–0.25496	–0.23042	580	–0.15398	–0.14618	–0.10852
610	–0.253	–0.25257	–0.22543	610	–0.1512	–0.1427	–0.10446
640	–0.25282	–0.25084	–0.22129	640	–0.14854	–0.13943	–0.10079
670	–0.25328	–0.2498	–0.21795	670	–0.14603	–0.13637	–0.09746
700	–0.25447	–0.2495	–0.21537	700	–0.14369	–0.13353	–0.09444
730	–0.25649	–0.25001	–0.21353	730	–0.14151	–0.13091	–0.09169
760	–0.25945	–0.2514	–0.2124	760	–0.13952	–0.12851	–0.08919
790	–0.26353	–0.25379	–0.21199	790	–0.13772	–0.12633	–0.0869
820	–0.26893	–0.25732	–0.21233	820	–0.13611	–0.12435	–0.08481
850	–0.27596	–0.26219	–0.21347	850	–0.13469	–0.12259	–0.0829
880	–0.28506	–0.26868	–0.21547	880	–0.13345	–0.12102	–0.08115
910	–0.29689	–0.27722	–0.21843	910	–0.13239	–0.11964	–0.07955
940	–0.31248	–0.28844	–0.22253	940	–0.13145	–0.11843	–0.07808
970	–0.33342	–0.30328	–0.22797	970	–0.13042	–0.11732	–0.07673
1000	–0.31394	–0.31178	–0.23509	1000	–0.12821	–0.11612	–0.0755

Table 8

Density of temperature coefficients for atmospheric muons detected below the ground at depths of 20 and 40 m (Yakutsk) at zenith angles of 0, 30, and 60°, with absorption ranges of protons L=110 g/cm^2 and pions l=120 g/cm^2

Atmospheric depth, *h*, mbar	20 m w.e.			Atmospheric depth, *h*, mbar	40 m w.e.		
	0	30	60		0	30	60
40	–0.13358	–0.0958	0.28942	40	0.064703	0.158819	0.84925
70	–0.07302	–0.02386	0.292926	70	0.139429	0.234782	0.71185
100	–0.04169	0.003976	0.203583	100	0.16174	0.23924	0.49368
130	–0.02882	0.007893	0.111179	130	0.156512	0.2122	0.31057
160	–0.02684	–1.2E–05	0.039647	160	0.137747	0.17367	0.17981
190	–0.031	–0.01295	–0.00927	190	0.113267	0.13349	0.0932
220	–0.0386	–0.02746	–0.04016	220	0.087319	0.09617	0.03842
250	–0.04715	–0.04092	–0.05795	250	0.06285	0.064127	0.00521
280	–0.0551	–0.05206	–0.06696	280	0.041378	0.038034	–0.0140
310	–0.06189	–0.0607	–0.07048	310	0.023285	0.017489	–0.0246
340	–0.06735	–0.067	–0.07076	340	0.008461	0.001717	–0.0300
370	–0.0715	–0.07132	–0.06923	370	–0.00342	–0.01013	–0.0323
400	–0.07448	–0.07402	–0.06677	400	–0.01276	–0.01882	–0.0328
430	–0.07643	–0.07543	–0.06391	430	–0.01996	–0.02506	–0.0323
460	–0.07753	–0.07585	–0.06096	460	–0.0254	–0.02939	–0.0313
490	–0.07794	–0.07554	–0.05807	490	–0.02941	–0.03228	–0.0301
520	–0.0778	–0.0747	–0.05534	520	–0.03229	–0.03408	–0.0288
550	–0.07724	–0.07348	–0.05279	550	–0.03426	–0.03508	–0.0276
580	–0.07636	–0.07202	–0.05044	580	–0.03553	–0.03549	–0.0263
610	–0.07525	–0.07041	–0.04828	610	–0.03625	–0.03547	–0.0252
640	–0.07399	–0.06872	–0.0463	640	–0.03656	–0.03517	–0.0241
670	–0.07262	–0.06699	–0.04449	670	–0.03655	–0.03465	–0.0231
700	–0.07119	–0.06528	–0.04282	700	–0.03631	–0.034	–0.0222
730	–0.06973	–0.0636	–0.04129	730	–0.0359	–0.03327	–0.0214
760	–0.06828	–0.06197	–0.03988	760	–0.03537	–0.03249	–0.0206
790	–0.06684	–0.0604	–0.03857	790	–0.03476	–0.03169	–0.0199
820	–0.06543	–0.0589	–0.03736	820	–0.03409	–0.03089	–0.0192
850	–0.06406	–0.05748	–0.03624	850	–0.03338	–0.03009	–0.0186
880	–0.06273	–0.05613	–0.0352	880	–0.03266	–0.02932	–0.0180
910	–0.06144	–0.05484	–0.03422	910	–0.03192	–0.02857	–0.0174
940	–0.06016	–0.05362	–0.03332	940	–0.03118	–0.02784	–0.0169
970	–0.05888	–0.05246	–0.03247	970	–0.03045	–0.02714	–0.0164
1000	–0.05754	–0.05134	–0.03167	1000	–0.02972	–0.02647	–0.0160