Solar-Terrestrial Physics. 2022. Vol. 8. Iss. 3. P. 46–50. DOI: 10.12737/stp-83202207. © 2022 Polyakov A.R. Published by INFRA-M Academic Publishing House

UDC 551.510.535
DOI: 10.12737/stp-83202207

Received February 15, 2022
Accepted June 15, 2022

STRUCTURE OF GROUPS OF EIGENFREQUENCIES IN SPECTRA OF GEOMAGNETIC PULSATIONS IN THE NIGHTSIDE MAGNETOSPHERE

A.R. Polyakov
Institute of Solar-Terrestrial Physics SB RAS,
Irkutsk, Russia, polar@iszf.irk.ru

Abstract. Using a new method of correlation function for amplitude and phase fluctuations (APCF), records of geomagnetic field component fluctuations (observatories Mondy and Borok) have been processed for a number of hour zones of the nightside magnetosphere. This method is meant to detect groups of equidistant frequencies inside the spectrum of source signal and also to measure a difference between two neighbor frequencies in each of these groups.

The groups of equidistant frequencies in broadband spectra of these fluctuations are shown to depend on eigenfrequencies of the 2D Alfvén wave resonator, just as for dayside fluctuations [Polyakov, 2018]. An empirical relation has been found between a combination of parameters of this resonator and local time. The similarity between structural elements in the final products of processing of Alfvén fluctuation N-S and E-W components for almost all hour zones clearly indicates the reliability of the results of the APCF method in processing any broadband signals.

Keywords: signal processing technique, correlation function, eigenfrequencies.

INTRODUCTION

The main result of the article series [Polyakov, 2010, 2014, 2015, 2018] should be considered the development of a new innovative signal processing method and its associated computer program. The method is based on the analysis of a specially constructed correlation function for amplitude and phase fluctuations (APCF) and is designed to detect groups of equidistant frequencies in the broadband spectrum of the source signal.

The APCF method is described in detail in [Polyakov, 2015, 2018]. This paper lists only its basic concepts. At the initial stage, a source signal is converted into a small addition to the sinusoid of a given frequency. For such a signal, deviations of the amplitude $n_i(\Theta_i)$ and phase $\gamma_i(\Theta_i)$ from the amplitude and phase of an ideal sinusoid are found at each time step i; Θ_i is the phase of this sinusoid.

At the next stage, the cross- and autocorrelation functions of amplitude and phase fluctuations are determined and used to calculate the function

$$G(\tau)=\left[\overline{\gamma(\Theta)\gamma(\Theta-\tau)}\right]\left[\overline{n(\Theta)n(\Theta-\tau)}\right]-$$
$$-\left[\overline{\gamma(\Theta)n(\Theta-\tau)}\right]\left[\overline{n(\Theta)\gamma(\Theta-\tau)}\right].$$

The overline means averaging over Θ; τ is the Θ phase shift. The correlation function $G(\tau)$ turned out to have one interesting property. In the works listed above, when processing simulated wave signals of various types in 1D and 2D resonators, as well as signals simply constructed as a sum of sinusoids of different frequencies, it has been shown that if there is an equidistant frequency group in the source signal spectrum, peaks periodically following each other along the τ-axis appear in the structure of the function G. From the position of the first peak τ_1, we can indirectly measure [Polyakov, 2018] the difference between two adjacent frequencies Δf of this equidistant group in the source signal spectrum.

At the last stage of the processing, all sequences of periodic peaks of the function $G(\tau)$ are identified, and for each of them the difference Δf is measured. The final product of the source signal processing is a histogram of these differences.

For the traditional Fourier spectrum it is commonly supposed that the presence of a statistically significant peak in its structure means that the signal contains quasi-monochromatic oscillations at the frequency of this peak. In the spectrum of the APCF method (histogram), each peak corresponds not to one but to a whole group of oscillations whose frequencies represent an equidistant sequence. Position of the peak along the Δf axis defines the difference between two adjacent frequencies, which is a common characteristic of the entire group. The APCF method is not able to determine individual frequencies, it is even impossible to figure out in which part of the frequency band they are located, and how many frequencies each group contains. At the same time, it appeared that if one or more consecutive frequencies are missing from the group, this slightly changes the corresponding periodic peaks of the function $G(\tau)$. The APCF method can successfully detect even such defective frequency groups.

For natural broadband oscillations, eigenfrequencies of resonators for those harmonic numbers at which they become equidistant should be considered as the equidistant frequency groups in the spectrum. There are many such resonators of different wave types in Earth's magnetosphere [Leonovich, Mazur, 2001; Zhu, Kivelson, 1989; Lee, Lysak, 1994; Ruohoniemi et al., 1991; Samson, Harrold, 1992; Takahashi et al., 2010]. Each 1D standing wave of a resonator forms one equidistant frequency group and corresponds to one peak in the Δ*f* histogram. In [Polyakov, 2018], the APCF method and associated computer program have first been applied to natural signals, which are short-period geomagnetic field fluctuations (SPF) recorded on the Earth surface. It turned out that in all cases the Δ*f* histograms include a large number of peaks that periodically follow each other along the horizontal axis. It has been qualitatively shown that this behavior of peaks fits exactly into the scheme of the structure of eigenfrequencies of a 2D standing Alfvén wave if a small transverse additive is taken into account in the dispersion relation. This paper continues the experiments on processing of discrete records of geomagnetic field fluctuations but for the nightside magnetosphere instead of the dayside one [Polyakov, 2018]. The main peculiarity here should be considered the comparison between the Δ*f* histograms of the N-S and E-W components since it can allow us to clear up such an important problem of the APCF method as the reality and reliability of the final processing product. The Δ*f* histograms should display not any random arbitrary or false peaks, but those that correspond to eigenfrequency groups in the spectrum.

RESULTS

As noted above, for processing by the APCF method the SPF data was used which had been obtained by similar induction magnetometers with an operating frequency range from 1 mHz to 10 Hz at the observatories Mondy (Φ=46.7°, Λ=173.6°, *L*=2.1) and Borok (Φ=53.9°, Λ=114.3°, *L*=2.9), where Φ and Λ are the corrected geomagnetic latitude and longitude; *L* is the magnetic shell number.

The data is presented in the discrete form; the time step is 0.1 s. In the APCF computer program, before source signal processing we have to set the frequency range within which equidistant eigenfrequency groups are detected. In this case, this range has boundaries from 0.2 to 2.2 Hz and corresponds to the high-frequency component of the SPF range.

The entire time array for each station has been divided into seven separate time intervals from 20–21 LT on April 10, 2000 to 02–03 LT on April 11, 2000. For each interval, the records of the N-S and E-W components were processed.

The final result of the processing by the APCF method is a Δ*f* histogram. One of them (Mondy Observatory) is given, as an example, in Figure 1, *b*. The magnetic field components and the local time of the hour zone are also shown in this Figure. Along the vertical axis, *n* is the number of real measurements of Δ*f* in the entire given range of values, n_0 is the total number of all attempts of these measurements. The ratio n/n_0 can serve as estimated probability of observing each of the Δ*f* values in the plot. The *f* and Δ*f* values are given in a dimensionless form as multiplied by the time step of the source signal ($f \to f\Delta t$).

This histogram displays a sequence of many distinct, isolated peaks. The line connecting bases of these peaks gradually drops to zero along the vertical axis from the beginning to the end of the range of Δ*f* values. Since we are interested only in the structure and location of the peaks themselves, it would make sense to get rid of slow and too small-scale fluctuations. To do this, we adopt a bandpass filtering procedure with specially selected boundary frequencies. The histogram filtered in this way is shown in panel *c*. Obviously, the peaks in it are more illustrative and more convenient for analysis than those in panel *b*.

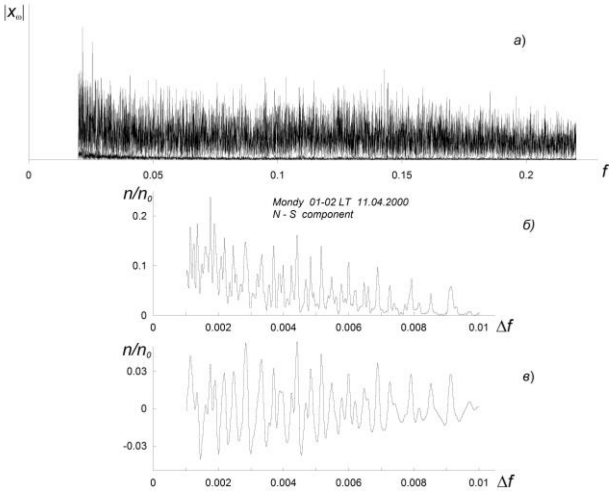

Figure 1. Example of Δ*f* histogram — a final product of the APCF processing technique for the N-S component of geomagnetic field fluctuations (*b*); traditional Fourier spectrum (*a*); the Δ*f* histogram after bandpass filtering (*c*)

In the spectrum of equidistant eigenfrequency groups (panel *b*), we can find 16 peaks with different Δ*f* values, from the peak with Δ*f*≈0.0018 to the peak with Δ*f*≈0.007, which follow each other along the horizontal axis at almost identical intervals. A similar feature of these peaks has been revealed in [Polyakov, 2018] for geomagnetic field fluctuations in the dayside magnetosphere. In this work, it was qualitatively shown that such a peak structure should be due to eigenfrequencies of a 2D standing Alfvén wave when the small transverse dispersion is considered [Leonovich, Mazur, 1987]. In this case, the eigenfrequencies are determined by two integers: *n* is the harmonic number along the magnetic field and *m* is the harmonic number along the transverse radial coordinate. To each *n* corresponds a group of equidistant frequencies with different *m*. The difference between two adjacent frequencies in such a group is determined by the ratio

$$\Delta f_n = \frac{\Omega_n}{2\pi} \frac{S\pi}{l_x}, \qquad (1)$$

where Ω_n is the frequency of longitudinal magnetic field oscillations (field line resonance); *S* is the Larmor radius of background protons, l_x is the size of the resonator along the radial coordinate.

Ratio (1) is best suited for interpreting the peaks mentioned above in Figure 1, *c*. At the same time, each peak has its own number *n* (there may be arbitrarily many such peaks, in this case there are 16 of them) and should be located along the Δ*f* axis equidistantly like the eigenfrequencies Ω_n for large harmonic numbers.

For comparison with the histogram in Figure 1, *a*, the traditional Fourier spectrum is presented in the frequency range set during processing. It is noticeable that there are no pronounced peaks in it at some individual frequencies. Such a spectrum is generally assumed to be similar to the spectrum of random noise and does not contain useful information. At the same time, the spectrum obtained by the APCF method (see Figure 1, *c*) shows that the Fourier spectrum should include peaks at frequencies of at least 16 equidistant groups with different Δf values. If each group contained 10 frequencies (this is the minimum estimate [Polyakov, 2018]), there would be 160 such peaks in total. Being located within the same frequency range, this set of peaks creates such, at first glance, a disordered structure of the spectrum which we can see in Figure 1, *a*.

Now let us compare the Δf histograms of the geomagnetic field fluctuation components. Figure 2 exhibits such histograms for two hour zones of the observatories Mondy (Figure 2, *a*) and Borok (Figure 2, *b*). In each case, a histogram of the N-S component is at the top, and below is a histogram of the E-W component.

In the N-S histogram of the left panel of Figure 2, *a* is a clear-cut sequence of ten distinct peaks of approximately the same height. Their relative position along the horizontal axis may be considered approximately equidistant. Obviously, these peaks, as well as those presented in Figure 1, *c*, are due to the differences between adjacent eigenfrequencies of 2D Alfvén wave (1). In the E-W histogram is also a sequence of peaks, although not as clear as in the N-S histogram since these peaks differ significantly in height.

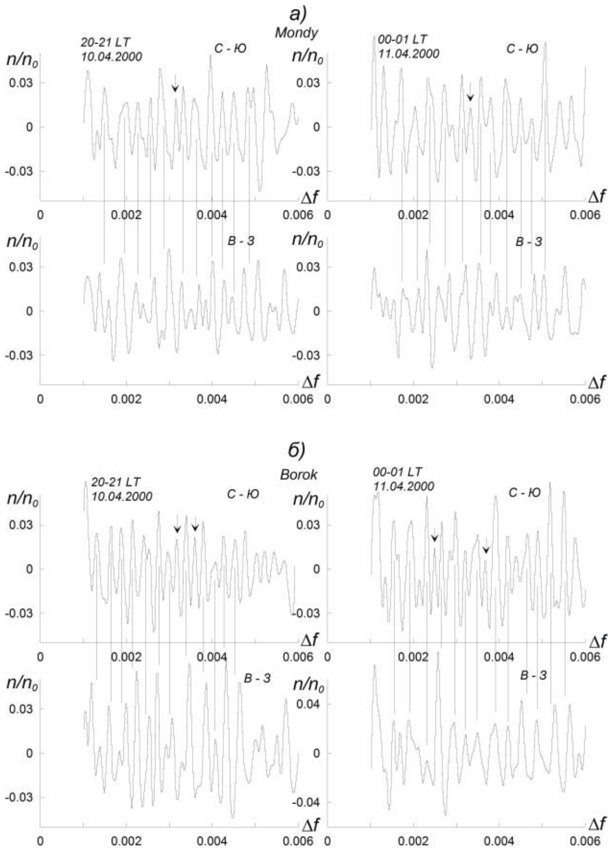

Figure 2. Comparison between positions of the peaks in the Δf histograms of the N-S and E-W components for some hour zones of the observatories Mondy and Borok

In order to compare the positions of the peaks along the Δf axis for the N-S and E-W histograms, vertical lines are drawn in the left panel of Figure 2, *a*, each passing through the peak maximum in the N-S histogram. For the first three lines on the left, the position of the lower ends is clearly seen to almost perfectly coincide with the position of tops of the corresponding peaks in the E-W histogram. On the fourth line, the top of the peak deviates noticeably, yet even in this case such a deviation is only 7 % of the Δf value at which the line crosses the horizontal axis. The next peak to the right in the N-S histogram (marked with a vertical arrow) does not have its analog in the E-W histogram. It appears, however, to be so close to the adjacent peak on the right that they should perhaps be considered one peak with a complex split structure. All other peaks in the N-S histogram in the series marked with vertical lines have a position that coincides almost exactly (the deviation does not exceed 5 %) with the position of the corresponding peaks in the E-W histogram.

The right panel of Figure 2, *a* for another hour zone exhibits a similar pattern. The lower ends of the vertical lines indicate that the positions of the peaks of the said series of the N-S histogram along the Δf axis coincide to a high accuracy with the position of the corresponding peaks of the E-W histogram. An exception is the only peak (marked with an arrow) of the 11 peaks of the entire series, which does not have a similar peak in the bottom histogram.

For Borok Observatory (see Figure 2, *b*), the relationship of the peaks in the Δf histograms with eigenfrequencies of 2D Alfvén waves (1) is most clearly manifested in the E-W component. This is especially noticeable in the plots of the right panel. The E-W histogram indicates that most of the peaks marked with vertical lines contain almost no small-scale fluctuations and are located along the Δf axis almost equidistantly. There are significantly more such fluctuations (interference) in the histogram of the right panel. The vertical lines both in the right panel and in the left one (Figure 2, *b*) bring out clearly, however, that the positions of the main peaks of the N-S and E-W histograms coincide almost perfectly, just as in Figure 2, *a*.

It is obvious that the groups of equidistant frequencies in the spectra of geomagnetic fluctuations recorded on the Earth surface should be determined mainly by the eigenfrequencies of Alfvén waves. In these waves, the frequency composition of the spectra should be the same for the N-S and E-W components.

The coincidence of the peak positions along the Δf axis for the histograms of different components in Figure 2 convincingly confirms this fact. At the same time, the recordings of fluctuations of different components for the APCF processing technique are signals independent of each other. Figure 2 shows that in all the cases presented the structure of the peaks of the final product of processing one of the signals sometimes coincides almost exactly with that of the peaks of the histogram of another signal. These coincidences represent a very important finding for further development of the APCF method of detecting equidistant frequency groups and its application to broadband signals. Now we can confidently say that this method and associated computer program provide objective and reliable information on the presence of equidistant frequency groups in the spectrum (group detection) and determine the Δ*f* parameters for each group with acceptable accuracy. At the beginning of the project, this did not seem obvious. It was assumed that interference in the composition of geomagnetic field fluctuations may also form equidistant frequency groups in the spectrum. The final histogram of the APCF method should contain a lot of random noise peaks that greatly distort the overall picture. Yet in real histograms (Figure 2) the contribution of such peaks is minimized; in most cases, we can see only equidistant peaks of eigenfrequencies of 2D magnetospheric standing Alfvén waves.

In the N-S and E-W histograms (Figure 2) for some hour zones, most of the peaks (marked with vertical lines) are located along the horizontal axis at approximately the same distance from each other. The vertical lines, like fence boards, look almost equidistant. From positions of these lines we can find the mean for the difference between Δf_{n+1} and Δf_n positions of two adjacent peaks. Denote this difference by δΔ*f*. Such δΔ*f* were measured for all processed hour zones of the geomagnetic field fluctuations recorded at the observatories Mondy and Borok. Figure 3 displays dependences of these measurements on local time for both observatories. The δΔ*f* difference is given in hertz, not in dimensionless units. This difference in the nightside magnetosphere is seen to be almost unchanged within a large interval of the azimuth coordinate from 20 to 3 LT. The mean of δΔ*f*≈3.2 mHz for the entire interval. Only in the hour interval 1–2 LT there is a slight increase to δΔ*f*≈3.5 mHz.

According to (1), the qualitative relation for δΔ*f* has the form

$$\delta\Delta f = \Delta f_{n+1} - \Delta f_n = \frac{\Delta\Omega}{2\pi}\frac{S\pi}{l_x}, \qquad (2)$$

where $\Delta\Omega=\Omega_{n+1}-\Omega_n$ for such harmonic numbers when the frequencies Ω_n become equidistant. It follows that the δΔ*f* characteristic measured is defined by three parameters of the right-hand side in (2). The above mentioned increase to δΔ*f*≈3.5 MHz is most likely due to a slight decrease in the radial size of the resonator l_x in the 1–2 LT sector.

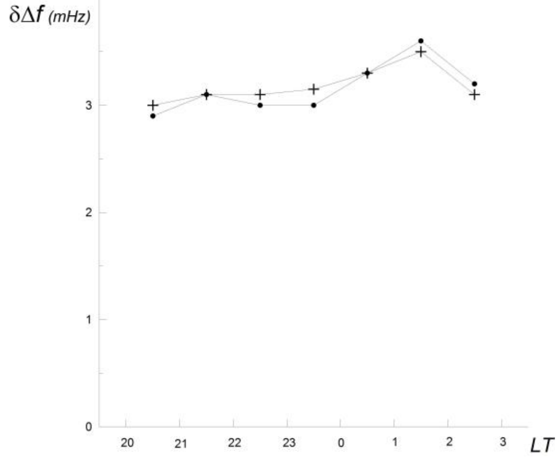

Figure 3. Dependence of the mean difference δΔ*f* between positions of adjacent peaks on local time (Mondy Observatory (dots), Borok Observatory (crosses))

Figure 3 clearly shows that all changes in the curve marked with dots (Mondy Observatory) match changes in the line marked with crosses (Borok Observatory). The maximum difference in δΔ*f* of these plots is only 5 %. Nonetheless, it should be taken into account that source signals for the processing are observed at different points of the Earth surface (observatories Mondy and Borok) at which geomagnetic field fluctuations are recorded by different instruments under conditions of different effects of natural and anthropogenic interference. That said, in Figure 3 we can see an almost exact coincidence of the δΔ*f* values for both observatories. This coincidence is another convincing evidence for authenticity and reliability of the results of the new APCF processing technique.

CONCLUSION

The following conclusions can be drawn.

1. Analysis of the Δ*f* histograms obtained using the new APCF signal processing technique shows that in all cases the nightside magnetosphere is characterized by the same periodic alternation of sequence of peaks along the Δ*f* axis as the peaks of histograms in the dayside magnetosphere [Polyakov, 2018]. The position of the peak *n* on the horizontal axis Δf_n is defined by (1) as the difference between two adjacent frequencies in an equidistant group, whose set forms eigenfrequencies of 2D standing Alfvén waves.

2. The N-S and E-W histograms of each hour zone have been used to measure means of the difference between positions of adjacent peaks along the horizontal axis of δΔ*f*. The local time dependences of this parameter for the observatories Mondy and Borok (see Figure 3) show that its values remain unchanged within a large-scale azimuth sector of the nightside magnetosphere. The mean of δΔ*f*=3.2 mHz in this sector. The difference δΔ*f* is shown to depend on a combination of three parameters of 2D Alfvén wave resonator (2). Small deviations from the mean in Figure 3 must be associated with a change in the resonator's radial size l_x.

3. Comparing histograms for the fluctuations of the N-S and E-W components (see Figure 2) indicates that in each such pair we can see similar peaks whose positions coincide to a high accuracy along the horizontal axis. This confirms the hypothesis that these peaks originate from eigenfrequency groups of 2D Alfvén wave. At the same time, such, in some cases almost exact, coincidence of the peaks by the components strongly suggests that the new APCF processing technique can provide us with consistent and reliable information on equidistant frequency groups in spectra of broadband signals. This important finding of this work is also confirmed by the coincidence of changes in the δΔ*f* dependences on LT (see Figure 3) for the observatories Mondy and Borok since source signals for the processing were recorded at a great distance from each other under conditions with different levels of instrument and anthropogenic interference.

The work was financially supported by the Ministry of Science and Higher Education of the Russian Federation.

The results were obtained using the equipment of Shared Equipment Center «Angara» [http://ckp-rf.ru/ckp/3056].

REFERENCES

Lee D.-H., Lysak R.L. Numerical studies on ULF wave structures in the dipole model, in: Solar Wind Sources of Magnetospheric Ultra-Low-Frequency Waves. *Geophys. Monograph Ser*. 1994, vol. 81, pp. 293–297.

Leonovich A.S., Mazur V.A. Dynamics of small-scale Alfvén waves in magnetospheric resonator. *Fizika plazmy* [Plasma Phys.]. 1987, vol. 13, iss. 7, pp. 800–810. (In Russian).

Leonovich A.S., Mazur V.A. On the spectrum of magnetosonic eigenoscillations of an axisymmetric magnetosphere. *J. Geophys. Res*. 2001, vol. 106, no. A3, 3919.

Polyakov A.R. New method of processing records of seismic oscillations based on analysis of correlation functions of random amplitude and phase fluctuations. Parts 1–2. *Solnechno-zemnaya fizika* [Solar-Terr. Phys.]. 2010, iss. 15, pp. 44–57. (In Russian).

Polyakov A.R. Standing MHD wave structure in the magnetosphere analyzed using the method of correlation functions of amplitude and phase fluctuations. *Geomagnetism and Aeronomy*. 2014, vol. 54, no. 6, pp. 735–745. DOI: 10.1134/S0016793214060140.

Polyakov A.R. Relationship of peaks of correlation functions of amplitude and phase fluctuations with eigenfrequencies in oscillation spectrum. *Solnechno-zemnaya fizika* [Solar-Terr. Phys.]. 2015, vol. 1, no. 3, pp. 62–71. (In Russian). DOI: 10.12737/10455.

Polyakov A.R. Detecting groups of equidistant frequencies in spectra of geomagnetic pulsations. *Solar-Terr. Phys*. 2018, vol. 4, iss. 4, pp. 33–41. DOI: 10.12737/stp-44201805.

Ruohoniemi J.M., Greenwald R.A., Baker K.B. HF radar observations of Pc5 field line resonances in the midnight/early morning MLT sector. *J. Geophys. Res*. 1991, vol. 96, iss. A9, pp. 15697–15710. DOI: 10.1029/91JA00795..

Samson J.C., Harrold B.G. Field line resonances associated with waveguides in the magnetosphere. *Geophys. Res. Lett*. 1992, vol. 19, iss. 5, pp. 441–444. DOI: 10.1029/92gl00116.

Takahashi K., Bonnell J., Glassmeier K.-H., Angelopoulos V., Singer H.J., Chi P.J., Denton R.E., Nishimura Y., Lee D.-H., Nosé M., Liu W. Multipoint observation of fast mode waves trapped in the dayside plasmasphere. *J. Geophys. Res*. 2010, vol. 115, iss. A15, A12247. DOI: 10.1029/2010JA015956.

Zhu X., Kivelson M.G. Global mode ULF pulsations in a magnetosphere with a nonmonotonic Alfvén velocity profile. *J. Geophys. Res*. 1989, vol. 94, iss. A2, pp. 1479–1485.DOI: 10.1029/ja094ia02p01479.

URL: http://ckp-rf.ru/ckp/3056 (accessed March 30, 2022).

Original Russian version: Polyakov A.R., published in Solnechno-zemnaya fizika. 2022. Vol. 8. Iss. 3. P. 49–53. DOI: 10.12737/szf-83202207.

How to cite this article

Polyakov A.R. Structure of groups of eigenfrequencies in spectra of geomagnetic pulsations in the nightside magnetosphere. *Solar-Terrestrial Physics*. 2022. Vol. 8. Iss. 3. P. 46–50. DOI: 10.12737/stp-83202207.